Sup YouTubers! Blaze is back to show you another couple of math problems that I found at: http://bloq.prepscholar.com/hardest-sat-math-questions

Want to test yourself against the most difficult SAT math questions? Want to know what makes these questions so difficult and how best to solve them? If you're ready to really sink your teeth into the SAT math section and have your sights set on that perfect score, then this is the guide for you.

We've put together what we believe to be the 13 most difficult questions for the new 2016 SAT, with strategies and answer explanations for each. These are all hard SAT math questions from College Board SAT practice tests, which means understanding them is one of the best ways to study for those of you aiming for perfection.

## Brief Overview of the SAT Math Section

The SAT math section is broken into two subsections (always the third and fourth sections of the test). The first math subsection (labeled as " 3 ") does NOT allow you to use a calculator; the second math subsection (labeled as " 4 ") does allow the use of a calculator. Don't fear: if it doesn't allow for a calculator, it means you don't need a calculator to answer the question.

Each math subsection is arranged in order of ascending difficulty, so on each subsection, question 1 will be "easy" and question 15 will be considered "difficult." However, the ascending difficulty resets from easy to hard on the grid-ins. Hence, the multiple choice will be arranged in increasing difficulty (questions 1 and 2 being the easiest, questions 14 and 15 being the hardest) and then the difficulty level resets for the grid-in section (meaning questions 16 and 17 will again be "easy" and questions 19 and 20 will be very difficult).

The way the SAT classifies "difficulty" is by how long it takes an average student to solve a problem, as well as the percentage of students who answer the question correctly. As you might expect, the longer it takes to solve a problem and the fewer people who answer it correctly, the more difficult the problem.

So, with very few exceptions, the most difficult SAT math problems will be clustered in the far end of the multiple choice segments or the later half of the grid-in questions. Other than their placement on the test, these questions share a few other
commonalities. Let us look at what these types of questions have in common, then look at example questions and how to solve them.

## But First, Should You Be Focusing on the Hardest Math Questions Right Now?

If you're just getting started in your study prep (or if you've simply skipped this first, crucial step), definitely stop and take a full practice test to gauge your current scoring level. Check out our guide to all the free SAT practice tests available online and then sit down to take a test all at once. The absolute best way to assess your current level is to simply take the SAT practice test as if it were real, keeping strict timing and working straight through with only the allowed breaks (we know--probably not your favorite way to spend a Saturday).

Once you've got a good idea of your current level and percentile ranking, you can set milestones and goals for your ultimate score. If you're currently scoring in the 200400 range on the Math section or the 400-600 range, your best bet is first to check out our guides to improving your math score to where you want it to be before you start in trying to tackle the most difficult math problems on the test.

If, however, you're already scoring above a 600 on the Math section and want to test your mettle for the real SAT, then definitely proceed to the rest of this guide. If you're aiming for perfect (or close to), then you'll need to know what the most difficult SAT math questions look like and how to solve them. And luckily, that's exactly what we'll do.

WARNING: since there are a limited number of official SAT practice tests, you may want to wait to read this article until you've attempted all or most of the first four official practice tests (since the questions below were taken from those tests). If you're worried about spoiling those tests, stop reading this guide now; come back and read it when you've completed them.

## The 13 Hardest SAT Math Questions No Calculator Questions

## Question 1

ANSWER EXPLANATION: If you think of the equation as an equation for a line
$y=m x+b$
where
$\mathrm{C}=5 / 9$ ( $\mathrm{F}-32$ )
or
$\mathrm{C}=(5 / 9) \mathrm{F}-(5 / 9) 32$
you can see the slope of the graph is $5 / 9$, which means that for an increase of 1 degree Fahrenheit, the increase is $5 / 9$ of 1 degree Celsius. Therefore, statement I is true. This is the equivalent to saying that an increase of 1 degree Celsius is equal to an increase of 9/5 degrees Fahrenheit. Since 9/5 $=1.8$, statement II is true. On the other hand, statement III is not true, since a temperature increase of $9 / 5$ degrees Fahrenheit, not 5/9 degree Fahrenheit, is equal to a temperature increase of 1 degree Celsius. The final answer is D.

ANSWER EXPLANATION: There are two ways to solve this question. The faster way is to multiply each side of the given equation by ax-2 (so you can get rid of the fraction). When you multiply each side by ax-2, you should have:

$$
C=\frac{5}{9}(F-32)
$$

The equation above shows how a temperature $F$, measured in degrees Fahrenheit, relates to a temperature C, measured in degrees Celsius. Based on the equation, which of the following must be true?
I. A temperature increase of 1 degree Fahrenheit is equivalent to a temperature
increase of $\frac{5}{9}$ degree Celsius.
II. A temperature increase of 1 degree Celsius is equivalent to a temperature increase of 1.8 degrees Fahrenheit.
III. A temperature increase of $\frac{5}{9}$ degree

Fahrenheit is equivalent to a temperature increase of 1 degree Celsius.
A) I only
B) II only
C) III only
D) I and II only

## Question 2

The equation $\frac{24 x^{2}+25 x-47}{a x-2}=-8 x-3-\frac{53}{a x-2}$ is true for all values of $x \neq \frac{2}{a}$, where $a$ is a constant.

What is the value of $a$ ?
A) -16
B) -3
C) 3
D) 16

$$
24 x^{2}+25 x-47=(-8 x-3)(a x-2)-53
$$

You should then multiply ( $-8 x-3$ ) and (ax-2) using FOIL.

$$
24 x^{2}+25 x-47=-8 a x^{2}-3 a x+16 x+6-53
$$

Then, reduce on the right side of the equation

$$
24 x^{2}+25 x-47=-8 a x^{2}-3 a x+16 x-47
$$

Since the coefficients of the x2-term have to be equal on both sides of the equation, $-8 a=24$, or $a=-3$.

The other option which is longer and more tedious is to attempt to plug in all of the answer choices for a and see which answer choice makes both sides of the equation equal. Again, this is the longer option, and I do not recommend it for the actual SAT as it will waste too much time. The final answer is $B$.

If $3 x-y=12$, what is the value of $\frac{8^{x}}{2^{y}}$ ?
A) $2^{12}$
B) $4^{4}$
C) $8^{2}$
D) The value cannot be determined from the information given.

## Question 3

ANSWER EXPLANATION: One approach is to express

$$
8^{x} / 2^{y}
$$

so that the numerator and denominator are expressed with the same base. Since 2 and 8 are both powers of 2 , substituting $2^{3}$ for 8 in the numerator of $8^{x} / 2^{y}$ gives

$$
\left(2^{3}\right)^{x} / 2^{y}
$$

which can be rewritten

$$
2^{3} x / 2^{y}
$$

Since the numerator and denominator of have a common base, this expression can be rewritten as $2(3 x-y)$. In the question, it states that $3 x-y=12$, so one can substitute 12 for the exponent, $3 x-y$, giving that the

$$
8^{x} / 2^{y}=2^{12}
$$

The final answer is $A$.

## Question 4

$$
\frac{8-i}{3-2 i}
$$

If the expression above is rewritten in the form $a+b i$, where $a$ and $b$ are real numbers, what is the value of $a$ ? (Note: $i=\sqrt{-1}$ )
A) 2
B) $\frac{8}{3}$
C) 3
D) $\frac{11}{3}$

## Question 5

In triangle $A B C$, the measure of $\angle B$ is $90^{\circ}$,
$B C=16$, and $A C=20$. Triangle $D E F$ is similar to
triangle $A B C$, where vertices $D, E$, and $F$
correspond to vertices $A, B$, and $C$, respectively, and
each side of triangle DEF is $\frac{1}{3}$ the length of the
corresponding side of triangle $A B C$. What is the

## ANSWER EXPLANATION:

To rewrite 8-i/3-2i in the standard form a+bi, you need to multiply the numerator and denominator of 8-i/3-2i by the conjugate, $3+2 \mathrm{i}$. This equals

$$
(8-i / 3-2 i)(3+2 i / 3+2 i)=
$$

$$
24+16 i-3+(-i)(2 i) /\left(3^{2}\right)+(2 i)^{2}
$$

Since $i^{2}=-1$, this last fraction can be reduced simplified to
$24+16 i-3 i+2 / 9-(-4)=26+13 i / 13$
which simplifies further to $2+\mathbf{i}$. Therefore, when $8-i / 3-2 i$ is rewritten in the standard form $a+b i$, the value of $a$ is 2 . The final answer is $A$.

ANSWER EXPLANATION: Triangle $A B C$ is a right triangle with its right angle at $B$. Therefore, $\overline{A C}$ is the hypotenuse of right triangle $A B C$, and $\overline{A B}$ and $\overline{B C}$ are the legs of right triangle ABC. According to the Pythagorean theorem,
value of $\sin F$ ?

$$
A B=\sqrt{ } 202-162=\sqrt{ } 400-256=\sqrt{ } 144=12
$$

Since triangle DEF is similar to triangle $A B C$, with vertex $F$ corresponding to vertex $C$, the measure of angle $\angle \mathrm{F}$ equals the measure of angle $\angle \mathrm{C}$. Therefore, $\sin F=\sin C$. From the side lengths of triangle ABC,
$\operatorname{sinF}=0$ pposite side/ hypotenuse $=A B / A C=12 / 20=3 / 5$
Therefore, $\sin F=3 / 5$. The final answer is $3 / 5$ or 0.6.
Calculator Questions

## Question 6

|  | Handedness |  |
| :--- | :---: | :---: |
| Gender | Left | Right |
| Female |  |  |
| Male |  |  |
| Total | 18 | 122 |

The incomplete table above summarizes the number of left-handed students and right-handed students by gender for the eighth-grade students at Keisel Middle School. There are 5 times as many right-handed female students as there are left-handed female students, and there are 9 times as many right-handed male students as there are left-handed male students. If there is a total of 18 left-handed students and 122 right-handed students in the school, which of the following is closest to the probability that a right-handed student selected at random is female? (Note: Assume that none of the eighth-grade students are both right-handed and left-handed.)
A) 0.410
B) 0.357
C) 0.333
D) 0.250

ANSWER EXPLANATION: In order to solve this problem, you should create two equations using two variables ( $x$ and $y$ ) and the information you're given. Let $x$ be the number of lefthanded female students and let $y$ be the number of lefthanded male students. Using the information given in the problem, the number of righthanded female students will be $5 x$ and the number of right-handed male students will be $9 y$. Since the total number of left-handed students is 18 and the total number of right-handed students is 122 , the system of equations below must be true:

$$
\begin{gathered}
x+y=18 \\
5 x+9 y=122
\end{gathered}
$$

When you solve this system
of equations, you get $x=10$ and $y=8$. Thus, 50 of the 122 right-handed students are female. Therefore, the probability that a right-handed student selected at random is female is $50 / 122$, which to the nearest thousandth is 0.410 . The final answer is $A$.

Questions 7 \& 8
Use the following information for both question 7 and question 8.

If shoppers enter a store at an average rate of $r$ shoppers per minute and each stays in the store for an average time of $T$ minutes, the average number of shoppers in the store, $N$, at any one time is given by the formula $N=r T$. This relationship is known as Little's law.

The owner of the Good Deals Store estimates that during business hours, an average of 3 shoppers per minute enter the store and that each of them stays an average of 15 minutes. The store owner uses Little's law to estimate that there are 45 shoppers in the store at any time.

## Question 7

Little's law can be applied to any part of the store, such as a particular department or the checkout lines. The store owner determines that, during business hours, approximately 84 shoppers per hour make a purchase and each of these shoppers spend an average of 5 minutes in the checkout line. At any time during business hours, about how many shoppers, on average, are waiting in the checkout line to make a purchase at the Good Deals Store?

## ANSWER

EXPLANATION: Since the question states that Little's law can be applied to any single part of the store (for example, just the checkout line, then the average number of shoppers, N , in the checkout line at any time is $N=r t$, where $r$ is the number of shoppers entering the checkout line per minute and $T$ is the average number of minutes each shopper spends in the checkout line. Since 84 shoppers per hour make a purchase, 84 shoppers per hour enter the checkout line. However, this needs to be converted to the number of shoppers per minute (in order to be used with $t=5$ ). Since there are 60 minutes in one hour, the rate is 84 shoppers per hour/ 60 minutes $=1.4$ shoppers per minute. Using the given formula with $r=1.4$ and $t=5$ yields

$$
N=r t=(1.4)(5)=7
$$

Therefore, the average number of shoppers, N , in the checkout line at any time during business hours is 7 . The final answer is 7.

## Question 8

The owner of the Good Deals Store opens a new store across town. For the new store, the owner estimates that, during business hours, an average of 90 shoppers per hour enter the store and each of them stays an average of 12 minutes. The average number of shoppers in the new store at any time is what percent less than the average number of shoppers in the original store at any time? (Note: Ignore the percent symbol when entering your answer. For example, if the answer is $42.1 \%$, enter 42.1)

ANSWER EXPLANATION: According to the original information given, the estimated average number of shoppers in the original store at any time is 45 . In the question, it states that, in the new store, the manager estimates that an average of 90 shoppers per hour enter the store, which is equivalent to 1.5 shoppers per minute. The manager also estimates that each shopper stays in the store for an average of 12 minutes. Thus, by Little's law, there are, on average, $N=r t=(1.5)(12)=18$ shoppers in the new store at any time. This is

$$
(45-18 / 45) \times 100=60
$$

percent less than the average number of shoppers in the original store at any time. The final answer is 60.


A grain silo is built from two right circular cones and a right circular cylinder with internal measurements represented by the figure above. Of the following, which is closest to the volume of the grain silo, in cubic feet?
A) 261.8
B) 785.4
C) 916.3
D) $1,047.2$

## Question 9

ANSWER EXPLANATION: The volume of the grain silo can be found by adding the volumes of all the solids of which it is composed (a cylinder and two cones). The silo is made up of a cylinder (with height 10 feet and base radius 5 feet) and two cones (each with height 5 ft and base radius 5 ft ). The formulas given at the beginning of the SAT Math section:

Volume of a Cone

$$
V=1 / 3 \pi r^{2} h
$$

Volume of a Cylinder

$$
V=\pi r^{2} h
$$

can be used to determine the total volume of the silo. Since
the two cones have identical dimensions, the total volume, in cubic feet, of the silo is given by

$$
V_{\text {sillo }}=\Pi\left(5^{2}\right)(10)+(2)(1 / 3) \Pi\left(5^{2}\right)(5)=(4 / 3)(250) \pi
$$

which is approximately equal to $1,047.2$ cubic feet. The final answer is $D$.

## Question 10

If $x$ is the average (arithmetic mean) of $m$ and 9 , $y$ is the average of $2 m$ and 15 , and $z$ is the average of 3 m and 18 , what is the average of $x, y$, and $z$ in terms of $m$ ?
A) $m+6$
B) $m+7$
C) $2 m+14$
D) $3 m+21$

## ANSWER EXPLANATION:

Since the average (arithmetic mean) of 2 numbers is equal to the sum of the 2 numbers divided by 2 , the equations $\mathrm{x}=$ $m+9 / 2, y=2 m+15 / 2, z=$ $3 m+18 / 2$ are true. The average of $x, y$, and $z$ is given by $x+y+z / 3$. Substituting the expressions in $m$ for each variable ( $x, y, z$ ) gives

$$
[(m+9 / 2)+(2 m+15 / 2)+(3 m+18 / 2)] / 3
$$

This fraction can be simplified to $m+7$. The final answer is $\mathbf{B}$.

## Question 11

ANSWER EXPLANATION: The equation $f(x)=k$ gives the solutions to the system of equations

$$
y=f(x)=x^{3}-x^{2}-x-11 / 4
$$

and

$$
y=k
$$

A real solution of a system of two equations corresponds to a point of intersection of the graphs of the two equations in the xy-plane. The graph of $y=k$ is a horizontal line that contains the point $(0, k)$, and the line with equation $y=-3$ is a horizontal line that intersects the graph of


The function $f(x)=x^{3}-x^{2}-x-\frac{11}{4}$ is graphed in the $x y$-plane above. If $k$ is a constant such that the equation $f(x)=k$ has three real solutions, which of the following could be the value of $k$ ?
A) 2
B) 0
C) -2
D) -3
the cubic equation three times. Therefore, the equation $f(x)=-3$ has three real solutions. The final answer is $D$.

## Question 12

$$
q=\frac{1}{2} n v^{2}
$$

The dynamic pressure $q$ generated by a fluid moving with velocity $v$ can be found using the formula above, where $n$ is the constant density of the fluid. An aeronautical engineer uses the formula to find the dynamic pressure of a fluid moving with velocity $v$ and the same fluid moving with velocity $1.5 v$. What is the ratio of the dynamic pressure of the faster fluid to the dynamic pressure of the slower fluid?

$$
V_{2}=1.5 v_{1}
$$

Given the equation $\mathrm{q}=1 / 2 \mathrm{nv}{ }^{2}$, substituting the dynamic pressure and velocity of the faster fluid gives $q_{2}=1 / 2 n\left(v_{2}\right)^{2}$. Since $v_{2}=1.5 v_{1}$, the expression $1.5 v_{1}$ can be substituted for $v_{2}$ in this equation, giving $q_{2}=1 / 2 n\left(1.5 v_{1}\right)^{2}$. By squaring 1.5 , you can rewrite the previous equation as

$$
\mathrm{Q}_{2}=(2.25)(1 / 2) \mathrm{n}\left(\mathrm{v}_{1}\right)^{2}=(2.25) \mathrm{q}_{1}
$$

Therefore, the ratio of the dynamic pressure of the faster fluid is

$$
q 2 / q 1=2.25 q_{1} / q_{1}=2.25
$$

The final answer is $\mathbf{2 . 2 5}$ or 9/4.

## Question 13

For a polynomial $p(x)$, the value of $p(3)$ is -2 .
Which of the following must be true about $p(x)$ ?
A) $x-5$ is a factor of $p(x)$.
B) $x-2$ is a factor of $p(x)$.
C) $x+2$ is a factor of $p(x)$.
D) The remainder when $p(x)$ is divided by $x-3$ is -2 .

ANSWER EXPLANATION: If the polynomial $p(x)$ is divided by $x-3$, the result can be written as

$$
p(x) / x-3=q(x)+(r / x-3)
$$

where $q(x)$ is a polynomial and $r$ is the remainder. Since $x-3$ is a degree 1 polynomial, the remainder is a real number. Therefore,
$p(x)$ can be rewritten as $p(x)=(x-3) q(x)+r$, where $r$ is a real number. In the question it said that $p(3)=-2$ so it must be true that

$$
-2=p(3)=(3-3) q(3)+r=(0) q(3)+r=r
$$

Therefore, the remainder when $p(x)$ is divided by $x-3$ is -2 . The final answer is $D$.

I hope I helped all of you who wanted to learn something new or study for your next math test. Enjoy this amazing math!

